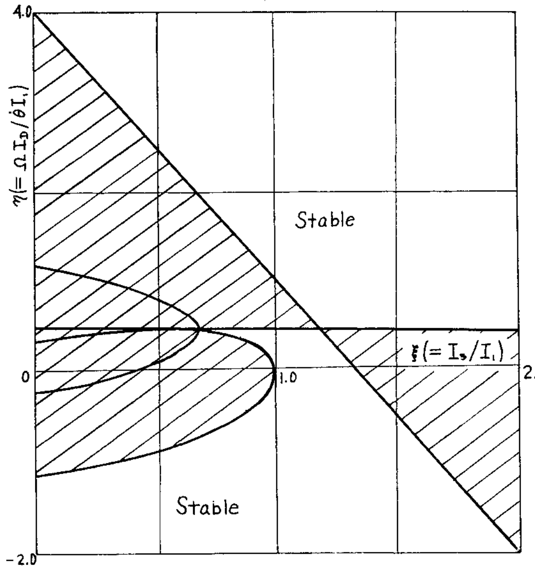

Fig. 2 Stability regions.

Stability Considerations

If Ω is constant, Eq. (9) shows that $\dot{\gamma}$ is also constant, and then the coefficients c_1, c_2, and c_3 are constant. Eliminating β between Eqs. (7) and (8) then leads to the differential equation

$$d^4\alpha/dt^4 + (c_1{}^2 - c_2 - c_3)d^2\alpha/dt^2 + c_2c_3\alpha = 0 \quad (13)$$

Thomson[2] has shown that solutions of this type of equation will remain small if the coefficients satisfy the inequalities

$$c_1{}^2 - c_2 - c_3 > 0 \quad (14)$$

$$c_2c_3 > 0 \quad (15)$$

$$(c_1{}^2 - c_2 - c_3)^2 - 4c_2c_3 > 0 \quad (16)$$

These relations thus represent the stability criteria for a satellite with a constant-speed disk gyro.

As a specific example, consider the case when $\dot{\gamma}$ is exactly equal to the negative of $\dot{\theta}$. This is a situation in which V essentially maintains a fixed attitude in inertial space. Under these conditions, the inequalities (14–16) may be written

$$3\xi - 1 + 4\eta(\eta - 1) > 0 \quad (17)$$

$$4 - 3\xi + 2\eta(2\eta - 5 + 3\xi) > 0 \quad (18)$$

$$9\xi^2 + (6 - 48\eta + 24\eta^2)\xi - 15 + 48\eta - 8\eta^2 - 32\eta^3 + 16\eta^4 > 0 \quad (19)$$

where ξ and η are dimensionless parameters given by

$$\xi = I_3/I_1 \qquad \eta = \Omega I_D/\dot{\theta}I_1 \quad (20)$$

The relations (17–19) determine regions of stability on an ξ-η diagram. These are shown in Fig. 2. Since ξ is essentially a function of the satellite geometry, and η is proportional to Ω, the diagram provides the gyro speed requirements necessary for maintaining stability for the various satellite shapes.

References

[1] Huston, R. L., "Gyroscopic stabilization of space vehicles," AIAA J. **1**, 1695–1696 (1963).

[2] Thomson, W. T., "Spin stabilization of attitude against gravity torque," J. Astronaut. Sci. **9**, 31–33 (1962).

[3] Kane, T. R. and Sobala, D., "A new method for attitude stabilization," AIAA J. **1**, 1365–1367 (1963).

[4] Nidey, R. A., "Gravitational torque on a satellite of arbitrary shape," ARS J. **30**, 203–204 (1960).

Optimization of Random Satellite Systems through the Use of Integer Programing Techniques

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A SATELLITE communication system that consists of randomly spaced satellites in circular orbit seems to be a likely choice for future satellite communications. A digital computer program has been developed to obtain the coverage given to any pair of ground stations by a satellite at any altitude and any inclination angle. The program requires only the latitude and longitude of the ground station, altitude, and inclination angle of the satellite, and ground station minimum antenna elevation angle; it can be utilized for both active and passive satellite systems. The assumption of a random satellite system permits some interesting manipulation, which allows us to obtain the optimum mix of satellites in inclination angle and altitude to satisfy specified communication requirements at minimum cost. For the randomness assumption to be correct, satellites must have random orbits for a given inclination, as well as be randomly phased with respect to each other within a given orbit.

Geometry of the Problem

For any two ground stations to be able to communicate, the same satellite must be visible to each. The region of communication for a particular ground station is defined by the antenna elevation angle B, the satellite orbit altitude H, and the latitude S and longitude M of the ground station. The ground antenna can rotate 360° in azimuth and (180°–2B) in elevation; it can see everything that is at least B° above the horizon (line-of-sight). The region of communication is the area formed by the intersection of the cone generated by the antenna with the sphere at altitude H. For circular orbits, the intersection of the antenna cone with the orbit sphere defines a circle, which can be radially projected to the earth, so that the region of communication can be defined in terms of latitude and longitude. Equation (1) defines the longitude boundaries of the region of communication for a specified latitude.[1,2]

$$L = M \pm \cos^{-1}\left[\frac{\sin A - \sin P \cos(90 - S)}{\cos P \sin(90 - S)}\right] \quad (1)$$

where $A = B + \sin^{-1}[R \cos B/(R + H)]$ and L is the longitude of the boundary of the region of communication for a specified latitude P. By next defining a time history of the ground track trajectory of a satellite at a particular altitude H and orbit inclination Q, we can tell what time a satellite will be in the region of communication for any ground station[3]:

$$X = \sin^{-1}(\sin Q \sin \omega t) \quad (2)$$

$$V = \tan^{-1}(\cos Q \tan \omega t) \quad (3)$$

$$Y = Y_0 + V - Et \quad (4)$$

where X and Y are latitude and longitude of the ground track, V the longitude change caused by orbital motion in the absence of earth rotation, E the earth angular rate, and ω the orbital angular velocity. Probabilities of visibility for the links are obtained by indexing the starting point of satellite ground track around the equator and averaging the percentage of time in view obtained at each position.

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Table 1 Assumed costs (millions of dollars)

H, naut miles	2000	4000	6000	8000	10,000
Example 1					
$Q = 0°$	10.0	10.0	10.0	...	...
$= 30°$	2.0	3.0	3.0	4.0	6.0
$= 45°$	2.0	3.0	4.0	6.0	10.0
$= 60°$	3.0	3.0	4.0	6.0	10.0
$= 90°$	3.0	4.0	6.0	6.0	10.0
Example 2					
$Q = 0°$	5.0	5.0	5.0	...	...
$= 30°$	1.1	1.8	2.0	2.33	3.0
$= 45°$	1.1	1.8	2.33	3.0	5.0
$= 60°$	1.125	2.0	2.33	3.0	5.0
$= 90°$	1.125	2.33	3.0	3.0	5.0

If P is the probability of seeing a satellite when only one is in a given orbit, then the probability of seeing a satellite P_N, given N satellites distributed at random within the same orbit, is

$$P_N = 1 - (1 - P)^N \tag{5}$$

If multiple access in the satellite is assumed, then each link needs to have only one satellite within its useful spatial volume, but if multiple use is denied, the problem becomes more complex. Solution for the case with multiple access, however, should give a proper indication or proportion of satellites to be used in each specific orbit.

To design a system, the probability of seeing a satellite when the N satellites are divided into groups at various altitudes and inclination angles must be determined:

$$P_k = 1 - \prod_{i,j}(1 - P_{ijk})^{n_{ij}} \tag{6}$$

where k is the index of links (station pairs); i the index of altitudes; j the index of inclination angles; P_{ijk} the probability of link k seeing a satellite at altitude i and inclination angle j; n_{ij} the number of satellites randomly distributed in orbit at altitude i and inclination angle j;

$$\sum_{i,j} n_{ij} = N$$

and P_k is the probability of link k having a satellite available.

Minimization of System Cost

The cost of placing satellites within the possible altitudes and orbital inclination angles varies as a function of both these parameters as does the coverage afforded to each link being considered. The links are individually required to handle a certain amount (and generally a certain type) of traffic. To account for the varying requirements, a measure of effectiveness (MOE) is assigned to each link; it is defined as the probability that no satellite is available to link k. Thus the satellite overlay must provide a probability of outage which is less than or equal to the MOE assigned to each link forming part of the system: $1 - P_k \leqq A_k$, where A_k is the MOE for link k. Therefore

$$\prod_{i,j} (1 - P_{ijk})^{n_{ij}} \leqq A_k \tag{7}$$

for each link k.

The set of equations thus formed provides a set of system constraint equations which must be satisfied. A set of linear constraint equations may be obtained by taking logarithms of both sides of this set of constraint equations. Thus

$$\log \left[\prod_{i,j} (1 - P_{ijk})^{n_{ij}}\right] = \sum_{i,j} n_{ij} \log(1 - P_{ijk}) \leqq \log A_k \tag{8}$$

Let

$$\log(1 - P_{ijk}) = b_{ijk} \qquad \log A_k = a_k \qquad n_{ij} = X_{ij} \tag{9}$$

Then

$$\log \left[\prod_{i,j} (1 - P_{ijk})^{n_{ij}}\right] = \sum_{i,j} X_{ij} b_{ijk} \leqq a_k$$
$$i = 1,2, \ldots m \qquad j = 1,2, \ldots h \qquad k = 1,2, \ldots z \tag{10}$$

Then the matrix of linear constraint equations is

$$\begin{aligned} \sum_{ij} X_{ij} b_{ij1} &\leq a_1 \\ \sum_{ij} X_{ij} b_{ij2} &\leq a_2 \\ &\vdots \\ \sum_{ij} X_{ij} b_{ijz} &\leq a_z \end{aligned} \tag{11}$$

The cost of placing the equivalent satellite is $C_{ij} = rC_s + C_r$ where r is the number of satellites launched simultaneously, C_s the construction cost per satellite payload, C_r the cost of launch vehicle, and C_{ij} the cost of placement of a satellite into the ij orbit.

The general design problem for a satellite overlay to satisfy a number of links simultaneously may then be stated as a standard linear programing problem. Minimize

$$C_{11}X_{11} + C_{12}X_{12} + \ldots + C_{1h}X_{1h} + C_{21}X_{21} + \ldots + C_{mh}X_{mh}$$

subject to Eqs. (11). Since fractions of a satellite have no

Table 2 Configurations for a minimal cost satellite system derived from analysis of integer programing

MOE	Altitude, naut miles	Total number	Angle of inclination	\$ cost, in millions
0.001	2,000	100	3 @ 45°–97 @ 60°	112
	4,000	57	44 @ 45°– 2 @ 60°–11 @ 90°	109
	6,000	42	1 @ 30°–39 @ 60°– 2 @ 90°	99
	8,000	36	23 @ 30°–13 @ 90°	96
	10,000	39	39 @ 30°	117
0.01	2,000	67	4 @ 45°–63 @ 60°	75
	4,000	37	17 @ 45°–14 @ 60°– 6 @ 90°	73
	6,000	28	27 @ 60°– 1 @ 90°	66
	8,000	24	15 @ 30°– 9 @ 90°	62
	10,000	26	26 @ 30°	78
0.05	2,000	44	6 @ 45°–38 @ 60°	49
	4,000	25	20 @ 45°– 1 @ 60°– 4 @ 90°	47
	6,000	19	3 @ 30°–16 @ 60°	43
	8,000	16	11 @ 30°– 5 @ 90°	41
	10,000	17	17 @ 30°	51
0.10	2,000	34	6 @ 45°–28 @ 60°	38
	4,000	19	15 @ 45°– 4 @ 60°	36
	6,000	14	1 @ 30°–12 @ 60°– 1 @ 90°	33
	8,000	12	7 @ 30°– 5 @ 90°	31
	10,000	13	13 @ 30°	39

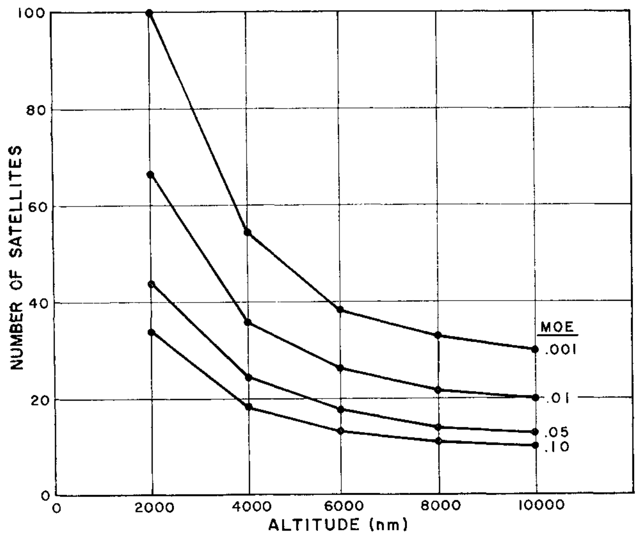

Fig. 1 Minimum number of satellites required as a function of altitude and desired measure of effectiveness. Note: Costs of achieving all orbits are equal.

meaning, the integer approach is attempted. In actuality, a linear programing solution will generally conform quite closely to the one obtained using integer programing techniques.

Example 1

The first problem selected for the purpose of illustrating the outlined technique of obtaining a minimum-cost satellite overlay consists of seven links that form a global belt of stations having a small variation in latitude. These links are: Japan $\xrightarrow{1}$ Midway $\xrightarrow{2}$ Hawaii $\xrightarrow{3}$ California $\xrightarrow{4}$ Washington, D. C. $\xrightarrow{5}$ Azores $\xrightarrow{6}$ France $\xrightarrow{7}$ Turkey. It is assumed that all links require the same MOE. That is, the satellite overlay will provide a probability of outage which is less than 0.01 ($A_k = 0.01$). Inclination angles of 0°, 30°, 45°, 60°, and 90° and discrete altitudes from 2000 to 10,000 naut miles at 2000-naut-mile intervals were chosen. Assumed cost values used in this problem are found in Table 1. Values for P_{ijk} were obtained using the technique outlined previously. The results for this example show a mix consisting of a single altitude and inclination. The optimum orbit is 6000 naut miles at 30° inclination. The system overlay required 35 satellites at a total cost of $105,000,000. Results obtained by linear programing rather than integer programing gave 34.78 satellites at an orbit of 6000 naut miles and 30° inclination; rounding this off, results are identical. This is not the case for example 2 below.

Example 2

Slightly different cost figures are assumed (see Table 1), and the orbit altitude is fixed at 6000 naut miles. The inclination angle was permitted to vary as in example 1. The P_{ijk} (for $H = 6000$ naut miles) and A_k are the same as those in example 1. The results show an optimum mix of 27 satellites at a 60° and one satellite in a polar orbit (90°) for a total cost of $66,000,000. The linear program results for this problem indicate 25.56 satellites at a 60° and 2.01 satellites at 90°. Roundoffs to the next highest integers which guarantee satisfaction of the constraint equation give 26 at 60° and 3 at 90°, which would raise the cost by $3.7,000,000 or 6% compared to the integer-program results. The cost estimates of example 2 were used in the integer program to obtain minmum cost mixes of satellites in various fixed altitude configurations; results are given in Table 2.

Example 3

The minimum number of satellites required to satisfy all coverage conditions simultaneously was also obtained. The method in this case assumes equal costs for placements of satellites. Under this assumption, minimum cost criteria will result in the minimum number of satellites for a specified MOE. Results are shown in Fig. 1 for each of the altitudes.

Conclusion

A method has been developed using integer programing techniques for optimizing a satellite configuration to satisfy global communication systems using minimum cost criteria. The technique may also be applied to any satellite overlay used to optimize presence of two satellites in the line-of-sight path of ground observers, as in navigational or satellite weather systems. Several examples have been presented, both for satisfying minimum cost criteria and for minimum number of satellites.

References

[1] Bennett, F. V., Coleman, T. L., and Houbolt, J. C., "Determination of the required number of randomly spaced communication satellites," NASA TN D-619 (January 1961).

[2] Bennett, F. V., "Further developments on the required number of randomly spaced communication and navigation satellites," NASA TN D-1020 (February 1962).

[3] Crickmay, C. J., "Optimizing the geographical location of satellite tracking stations," ARS J. **32**, 107–113 (1962).

[4] Levitan, R. E., "Integer programming 3, 7090 (PK IP93 and PK IPM3)," Share Distribution Document 1190 (September 1961).

Venus Swingby Mode for Manned Mars Missions

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A PROMISING new method for reducing Earth return velocities from Mars stopover or flyby missions is the Venus swingby mode, which utilizes the gravitational field of Venus to decelerate the spacecraft on the return leg, or to accelerate it on the outbound leg, to achieve a more favorable calendar date for return from Mars. Earth entry velocities are reduced from a maximum of 70,000 to 50,000 fps or less, without increases in mission propulsion requirements. Venus swingby modes have been examined previously by Ross[1] and others in connection with the 1970–1972 Mars-Venus dual planet flybys; it is the purpose of this note to show the general applicability of the Venus swingby technique to both Mars stopover and flyby missions for all mission opportunities (14 opportunities were examined, covering the period from 1971 to 1999).

Launch opportunities for Mars missions are associated with Mars-Earth oppositions, and precede by three to four months the opposition dates, which occur on the average every 26 months. Because of the eccentricity of the Mars orbit, the mission trajectory profiles change from one opposition to the

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